4c $2x^2+5x+k-5=0$

$a=2,\ b=5,\ c=k-5$

$b^2-4ac=5^2-4\times2\times(k-5)$

$=65-8k$

Exactly one solution so

$b^2-4ac=0\Rightarrow 65-8k=0$

$k=\frac{65}{8}$

5a $x^2+3x-3k=0$

$a=1,\ b=3,\ c=-3k$

$b^2-4ac=3^2-4\times1\times(-3k)$

$=9+12k$

Real solutions so

$b^2-4ac\geq0\Rightarrow 9+12k\geq0$

$k\geq-\frac{3}{4}$

5b $kx^2-7x+4=0$

$a=k,\ b=-7,\ c=4$

$b^2-4ac=(-7)^2-4\times k\times4$

$=49-16k$

Real solutions so

$b^2-4ac\geq0\Rightarrow 49-16k\geq0$

$k\leq\frac{49}{16}$

5c $-x^2+6x-k-2=0$

$a=-1,\ b=6,\ c=-k-2$

$b^2-4ac=6^2-4\times(-1)\times(-k-2)$

$=28-4k$

Real solutions so

$b^2-4ac\geq0\Rightarrow 28-4k\geq0$

$k\leq7$

6a $5x^2-x+2k=0$

$a=5,\ b=-1,\ c=2k$

$b^2-4ac=(-1)^2-4\times5\times2k$

$=1-40k$

No real solutions so

$b^2-4ac<0\Rightarrow 1-40k<0$

$k>\frac{1}{40}$

6b $-kx^2+4x+5=0$

$a=-k,\ b=4,\ c=5$

$b^2-4ac=4^2-4\times(-k)\times5$

$=16+20k$

No real solutions so

$b^2-4ac<0\Rightarrow 16+20k<0$

$k<-\frac{4}{5}$

6c $6x^2-5x+3-2k=0$

$a=6,\ b=-5,\ c=3-2k$

$b^2-4ac=(-5)^2-4\times6\times(3-2k)$

$=-47+48k$

No real solutions so

$b^2-4ac<0\Rightarrow -47+48k<0$

$k<\frac{47}{48}$

Try it 1F

1a $m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{8-7}{4-1}$

$=\frac{1}{3}$

1b $m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{6--2}{4-8}$

$=\frac{8}{-4}$

$=-2$

1c $m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-7-7}{-4--8}$

$=-\frac{14}{4}$

$=-\frac{7}{2}$

2a $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(7-5)^2+(4-2)^2}$

$=\sqrt{2^2+2^2}$

$=2\sqrt{2}$

2b $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-3-6)^2+(-1--4)^2}$

$=\sqrt{(-9)^2+3^2}$

$=3\sqrt{10}$

2c $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(4\sqrt{2}-\sqrt{2})^2+(-5-4)^2}$

$=\sqrt{(3\sqrt{2})^2+(-9)^2}$

$= 3\sqrt{11}$

3a Midpoint $=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

So $\left(\dfrac{1+2}{2},\dfrac{9+5}{2}\right)=(1.5, 7)$

3b Midpoint $=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

So $\left(\dfrac{-2+-5}{2},\dfrac{3+-7}{2}\right)=(-3.5, -2)$

3c Midpoint $=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

So $\left(\dfrac{6.4+-2.6}{2},\dfrac{-9.3+-3.7}{2}\right)=(1.9, -6.5)$

4a $y=8-2x$, gradient is –2, y-intercept is 8

4b $2y+x=3, 2y=3-x \Rightarrow y=\dfrac{3}{2}-\dfrac{1}{2}x$

Gradient is $-\dfrac{1}{2}$, y-intercept is $\dfrac{3}{2}$

4c $6x-9y-4=0,\ 9y=6x-4$

$y=\dfrac{2}{3}x-\dfrac{4}{9}$

Gradient is $\dfrac{2}{3}$, y-intercept is $-\dfrac{4}{9}$

5a Find the gradient:

$m=\dfrac{9-7}{2-3}$

$=\dfrac{2}{-1}$

$=-2$

$y-y_1=m(x-x_1)$ gives:

$y-7=-2(x-3)$ or $y-9=-2(x-2)$

So $y=-2x+13$

5b $m=\dfrac{5--1}{7-5}$

$=\dfrac{6}{2}$

$=3$

$y-y_1=m(x-x_1)$ gives:

$y+1=3(x-5)$ or $y-5=3(x-7)$

So $y=3x-16$

5c $m=\dfrac{2--4}{7--3}$

$=\dfrac{6}{10}$

$=\dfrac{3}{5}$

$y-y_1=m(x-x_1)$ gives:

$y+4=\dfrac{3}{5}(x+3)$ or $y-2=\dfrac{3}{5}(x-7)$

So $5y=3x-11$

6 Rearrange $l_1: 2y=3x-8$

$y=\dfrac{3}{2}x-4$

So $m=\dfrac{3}{2}$

$y-y_1=m(x-x_1)$ gives:

$y+2=\dfrac{3}{2}(x-3)$

$2y+4=3(x-3)$

$2y+4=3x-9$

$3x-2y-13=0$

7a Rearrange $3x+6y=2$

$6y=2-3x$

$y=\dfrac{1}{3}-\dfrac{1}{2}x$

Gradient is $-\dfrac{1}{2}$, so neither parallel nor perpendicular to $y=4-3x$ which has a gradient of –3.

7b Rearrange $5x-15y=7$

$15y=5x-7$

$y=\dfrac{1}{3}x-\dfrac{7}{15}$

Gradient is $\dfrac{1}{3}$ and $\dfrac{1}{3}\times(-3)=-1$ so perpendicular to $y=4-3x$

7c Rearrange $18x+6y+5=0$

$6y=-18x-5$

$y=-3x-\frac{5}{6}$

Gradient is –3 so parallel to $y=4-3x$

8 Rearrange $l_1: 6y=3-4x$

$y=\frac{1}{2}-\frac{2}{3}x$

so gradient is $-\frac{2}{3}$

Hence gradient of l_2 is $\frac{3}{2}$

$y-y_1=m(x-x_1)$ gives:

Equation $y-5=\frac{3}{2}(x+1)$

$2y-10=3(x+1)$

$2y-10=3x+3$

$3x-2y+13=0$

9 Midpoint is $\left(\frac{2+-12}{2},\frac{-3+5}{2}\right)=(-5,1)$

Gradient is $\frac{5--3}{-12-2}=-\frac{4}{7}$

So gradient of perpendicular bisector is $\frac{7}{4}$

since $-\frac{4}{7}\times\frac{7}{4}=-1$

$y-y_1=m(x-x_1)$ gives:

Equation $y-1=\frac{7}{4}(x+5)$

$7x-4y+39=0$

Bridging Exercise 1F

1a $m=\frac{8-7}{2-3}$

$=\frac{1}{-1}$

$=-1$

1b $m=\frac{-6--2}{-4-5}$

$=-\frac{4}{-9}$

$=\frac{4}{9}$

1c $m=\frac{-3.1-4.7}{2.6-1.3}$

$=-\frac{7.8}{1.3}$

$=-6$

1d $m=\frac{\frac{2}{3}-\frac{1}{3}}{\frac{3}{4}-\frac{1}{2}}$

$=\frac{\frac{1}{3}}{\frac{1}{4}}$

$=\frac{4}{3}$

1e $m=\frac{5-2}{2\sqrt{3}-\sqrt{3}}$

$=\frac{3}{\sqrt{3}}$

$=\sqrt{3}$

1f $m=\frac{5a-a}{a-3a}$

$=\frac{4a}{-2a}$

$=-2$

2a $d=\sqrt{(1-8)^2+(3-4)^2}$

$=\sqrt{(-7)^2+(-1)^2}$

$=5\sqrt{2}$

2b $d=\sqrt{(12--3)^2+(-7-9)^2}$

$=\sqrt{15^2+(-16)^2}$

$=\sqrt{481}$

2c $d=\sqrt{(-8.1-5.9)^2+(3.8-6.2)^2}$

$=\sqrt{14^2+(-2.4)^2}$

$=\sqrt{201.76}$

$=14.2$

2d $d=\sqrt{\left(\frac{3}{5}-\frac{1}{5}\right)^2+\left(-\frac{4}{5}--\frac{1}{5}\right)^2}$

$=\sqrt{\left(\frac{2}{5}\right)^2+\left(-\frac{3}{5}\right)^2}$

$=\frac{\sqrt{13}}{5}$

2e $d=\sqrt{(2-5)^2+(\sqrt{2}--3\sqrt{2})^2}$

$=\sqrt{3^2+(4\sqrt{2})^2}$

$=\sqrt{41}$

2f $d=\sqrt{(2k-k)^2+(-6k--3k)^2}$

$=\sqrt{k^2+(-3k)^2}$

$=k\sqrt{10}$

3a $\left(\frac{3+1}{2},\frac{9+7}{2}\right)=(2,\ 8)$

3b $\left(\frac{2+-3}{2},\frac{-4+-9}{2}\right)=(-0.5,\ -6.5)$

3c $\left(\frac{2.1+6.3}{2},\frac{3.5+-3.7}{2}\right)=(4.2,\ -0.1)$

3d $\left(\frac{\frac{2}{3}+-\frac{5}{3}}{2},\frac{-\frac{1}{2}+-\frac{3}{2}}{2}\right)=(-0.5,\ -1)$

3e $\left(\frac{6\sqrt{5}+-\sqrt{5}}{2},\frac{2\sqrt{5}+\sqrt{5}}{2}\right)=\left(\frac{5}{2}\sqrt{5},\frac{3}{2}\sqrt{5}\right)$

3f $\left(\frac{m+3m}{2},\frac{2n+-2n}{2}\right)=(2m,\ 0)$

4a $y=7x-4$

Gradient is 7, y-intercept is –4

4b Rearrange $y+2x=3$

$y=-2x+3$

Gradient is –2, y-intercept is 3

4c Rearrange $x-y=4$

$y=x-4$

Gradient is 1, y-intercept is –4

4d Rearrange $3x+2y=7$

$2y=7-3x$

$y=\frac{7}{2}-\frac{3}{2}x$

Gradient is $-\frac{3}{2}$, y-intercept is $\frac{7}{2}$

4e Rearrange $5x-2y=9$

$2y=5x-9$

$y=\frac{5}{2}x-\frac{9}{2}$

Gradient is $\frac{5}{2}$, y-intercept is $-\frac{9}{2}$

4f Rearrange $5y-3x=0$

$5y=3x$

$x=\frac{3}{5}x$

Gradient is $\frac{3}{5}$, y-intercept is 0

4g Rearrange $x+6y+3=0$

$6y=-3-x$

$y=-\frac{1}{2}-\frac{1}{6}x$

Gradient is $-\frac{1}{6}$, y-intercept is $-\frac{1}{2}$

4h Expand brackets and rearrange

$3(y-2)=4(x-1)$

$3y-6=4x-4$

$3y=4x+2$

$y=\frac{4}{3}x+\frac{2}{3}$

Gradient is $\frac{4}{3}$, y-intercept is $\frac{2}{3}$

5a $m=\frac{6-5}{0-2}$

$=\frac{1}{-2}$

$=-\frac{1}{2}$

$y-5=-\frac{1}{2}(x-2)$ or $y-6=-\frac{1}{2}(x-0)$

$y=-\frac{1}{2}x+6$

5b $m=\frac{-5--3}{2-1}$

$=\frac{-2}{1}$

$=-2$

$y+3=-2(x-1)$ or $y+5=-2(x-2)$

$y=-2x-1$

5c $m=\frac{-7-4}{7-4}$

$=-\frac{11}{3}$

$y-4=-\frac{11}{3}(x-4)$ or $y+7=-\frac{11}{3}(x-7)$

$11x+3y-56=0$

5d $m=\frac{-3--2}{4-8}$

$=\frac{-1}{-4}$

$=\frac{1}{4}$

$y+2=\frac{1}{4}(x-8)$ or $y+3=\frac{1}{4}(x-4)$

$y=\frac{1}{4}x-4$

5e $m=\frac{9--7}{5--3}$

$=\frac{16}{8}$

$=2$

$y+7=2(x+3)$ or $y-9=2(x-5)$

$y=2x-1$

5f $m=\frac{4\sqrt{2}--\sqrt{2}}{3\sqrt{2}-\sqrt{2}}$

$=\frac{5\sqrt{2}}{2\sqrt{2}}$

$=\frac{5}{2}$

$y+\sqrt{2}=\frac{5}{2}(x-\sqrt{2})$ or $y-4\sqrt{2}=\frac{5}{2}(x-3\sqrt{2})$

$y=\frac{5}{2}x-\frac{7}{2}\sqrt{2}$

6a $12y=-2x-3$

$y=-\frac{1}{6}x-\frac{1}{4}$

Gradient is $-\frac{1}{6}$ so perpendicular to $y=6x+5$ since $\left(-\frac{1}{6}\right)\times 6=-1$

6b $3y=2-18x$

$y=\frac{2}{3}-6x$

Gradient is –6 so neither parallel nor perpendicular to $y=6x+5$

6c $\frac{1}{2}y=3x+5$

$y=6x+10$

Gradient is 6 so parallel to $y=6x+5$

7a $16y=-24x-3$

$y=-\frac{3}{2}x-\frac{3}{16}$

Gradient is $-\frac{3}{2}$ so perpendicular to $y=\frac{2}{3}x-4$ since $\left(-\frac{3}{2}\right)\times\frac{2}{3}=-1$

7b $9y=-6x-2$

$y=-\frac{2}{3}x-\frac{2}{9}$

Gradient is $-\frac{2}{3}$ so neither parallel nor perpendicular to $y=\frac{2}{3}x-4$

7c $3y=2x-7$

$y=\frac{2}{3}x-\frac{7}{3}$

Gradient is $\frac{2}{3}$ so parallel to $y=\frac{2}{3}x-\frac{7}{3}$

8 Rearrange $12y=1-6x$

$y=\frac{1}{12}-\frac{1}{2}x$

so gradient is $-\frac{1}{2}$

8a Rearrange $2y=5-x$

$y=\frac{5}{2}-\frac{1}{2}x$

Gradient is $-\frac{1}{2}$ so parallel to $6x+12y=1$

8b Rearrange $9x=18y+4$

$18y=9x-4$

$y=\frac{1}{2}x-\frac{2}{9}$

Gradient is $\frac{1}{2}$ so neither parallel nor perpendicular to $6x+12y=1$

8c Rearrange $10x-5y+3=0$

$5y=10x+3$

$y=2x+\frac{3}{5}$

Gradient is 2 so perpendicular to $6x+12y=1$ since $2\times\left(-\frac{1}{2}\right)=-1$

9a Gradient of l_1 is 5

$y+3=5(x-3)$

$y+3=5x-15$

$5x-y-18=0$

9b Gradient of perpendicular is $-\frac{1}{5}$ since $-\frac{1}{5}\times 5=-1$

$y-1=-\frac{1}{5}(x+4)$

$-5y+5=x+4$

$x+5y-1=0$

10a Gradient of l_1 is $\frac{1}{2}$

$$y-5=\frac{1}{2}(x+1)$$
$$2y-10=x+1$$
$$x-2y+11=0$$

10b Gradient of perpendicular is –2 since $-2\times\frac{1}{2}=-1$

$$y-2=-2(x-6)$$
$$y-2=-2x+12$$
$$2x+y-14=0$$

11a Rearrange $l_1: y=9-3x$ so gradient of l_1 is –3

$$y+2=-3(x-8)$$
$$y+2=-3x+24$$
$$3x+y-22=0$$

11b Gradient of perpendicular is $\frac{1}{3}$ since $\frac{1}{3}\times(-3)=-1$

$$y+1=\frac{1}{3}(x+1)$$
$$3y+3=x+1$$
$$x-3y-2=0$$

12a Rearrange $l_1: 5y=-6x-2$

$$y=-\frac{6}{5}x-\frac{2}{5}$$

so gradient of l_1 is $-\frac{6}{5}$

$$y=-\frac{6}{5}(x-4)$$
$$5y=-6x+24$$
$$6x+5y-24=0$$

12b Gradient of perpendicular is $\frac{5}{6}$ since $\frac{5}{6}\times\left(-\frac{6}{5}\right)=-1$

$$y-3=\frac{5}{6}(x-12)$$
$$6y-18=5x-60$$
$$5x-6y-42=0$$

13a Rearrange $l_1: 2y=6x-1$

$$y=3x-\frac{1}{2}$$

so gradient of l_1 is 3

$$y-1=3\left(x-\frac{1}{2}\right)$$
$$y-1=3x-\frac{3}{2}$$
$$2y-2=6x-3$$
$$6x-2y-1=0$$

13b Gradient of perpendicular is $-\frac{1}{3}$ since $\left(-\frac{1}{3}\right)\times3=-1$

$$y+\frac{1}{2}=-\frac{1}{3}(x+1)$$
$$6y+3=-2(x+1)$$
$$6y+3=-2x-2$$
$$2x+6y+5=0$$

14a Midpoint is $\left(\frac{5+-3}{2},\frac{-7+5}{2}\right)=(1,-1)$

Gradient is $\frac{5--7}{-3-5}=\frac{12}{-8}$

$$=-\frac{3}{2}$$

So gradient of perpendicular bisector is

$m=\frac{2}{3}$ since $\frac{2}{3}\times\left(-\frac{3}{2}\right)=-1$

Equation is $y+1=\frac{2}{3}(x-1)$

$$2x-3y-5=0$$

14b Midpoint is $\left(\frac{-5+5}{2},\frac{-9+5}{2}\right)=(0,-2)$

Gradient is $\frac{5--9}{5--5}=\frac{14}{10}$

$$=\frac{7}{5}$$

So gradient of perpendicular bisector is

$m=-\frac{5}{7}$ since $\left(-\frac{5}{7}\right)\times\frac{7}{5}=-1$

Equation is $y+2=-\frac{5}{7}x$

$$5x+7y+14=0$$

14c Midpoint is $\left(\frac{-6+4}{2},\frac{2+12}{2}\right)=(-1,7)$

Gradient is $\frac{12-2}{4--6}=\frac{10}{10}$

$$=1$$

So gradient of perpendicular bisector is $m=-1$ since $(-1)\times 1=-1$

Equation is $y-7=-(x+1)$

$y=-x+6$

14d Midpoint is $\left(\frac{2+-1}{2}, \frac{-7+2}{2}\right)=\left(-\frac{1}{2}, -\frac{5}{2}\right)$

Gradient is $\frac{2--7}{-1-2}=\frac{9}{-3}$

$=-3$

So gradient of perpendicular bisector is $m=\frac{1}{3}$ since $\frac{1}{3}\times(-3)=-1$

Equation is $y+\frac{5}{2}=\frac{1}{3}\left(x+\frac{1}{2}\right)$

$x-3y-7=0$

14e Midpoint is $\left(\frac{-13+15}{2}, \frac{-5+-12}{2}\right)$

$=(1, -8.5)$

Gradient is $\frac{-12--5}{15--13}=\frac{-7}{28}$

$=-\frac{1}{4}$

So gradient of perpendicular bisector is $m=4$ since $4\times\left(-\frac{1}{4}\right)=-1$

Equation is $y+\frac{17}{2}=4(x-1)$

$8x-2y-25=0$

15a $y=5x-4$ and $y=3-2x$

$5x-4=3-2x$

$7x=7$

$x=1$

Substitute x value into either of the equations:

$y=5(1)-4=1$

So they intersect at (1, 1)

15b $y=8x$ and $y=3x-10$

$8x=3x-10$

$5x=-10$

$x=-2$

Substitute x value into either of the equations:

$y=8(-2)$

$=-16$

So they intersect at (–2, 16)

15c $y=7x-5$ and $y=-\frac{1}{2}x+5$

$7x-5=-\frac{1}{2}x+5$

$14x-10=-x-10$

$15x=20$

$x=\frac{4}{3}$

Substitute x value into either of the equations:

$y=7\left(\frac{4}{3}\right)-5$

$=\frac{13}{3}$

So they intersect at $\left(\frac{4}{3}, \frac{13}{3}\right)$

15d $y=\frac{1}{4}x+7$ and $y=5x-\frac{5}{2}$

$\frac{1}{4}x+7=5x-\frac{5}{2}$

$x+28=20x-10$

$19x=38$

$x=2$

Substitute x value into either of the equations:

$y=\frac{1}{4}(2)+7$

$=\frac{15}{2}$

So they intersect at $\left(2, \frac{15}{2}\right)$

16a $2x+3y=1$ and $3x-y=7$

Multiply second equation by 3: $9x-3y=21$

Add to first equation: $11x=22$

$x=2$

Substitute x value into either of the original equations:

$2(2)+3y=1$

$3y=-3$

$y=-1$

So they intersect at (2, –1)

16b $3x-2y=4$ and $x+y=8$

Multiply second equation by 3: $3x+3y=24$

Subtract first equation: $5y=20$

$$y=4$$
$$x+4=8$$
$$x=4$$

So they intersect at (4, 4)

16c $5x-7y=3$ and $2x+8y=3$

Multiply first equation by 2:
$10x-14y=6$ (1)
Multiply second equation by 5:
$10x+40y=15$ (2)

$(2)-(1): 54y=9$

$$y=\frac{1}{6}$$

Substitute y value into either of the original equations:

$$2x+8\left(\frac{1}{6}\right)=3$$
$$2x=\frac{5}{3}$$
$$x=\frac{5}{6}$$

So they intersect at $\left(\frac{5}{6},\frac{1}{6}\right)$

16d $-8x+5y=1$ and $3x+18y+7=0$

Multiply first equation by 3:
$-24x+15y=3$ (1)
Rearrange and multiply second equation by 8: $24x+144y=-56$ (2)

$(1)+(2): 159y=-53$

$$y=-\frac{1}{3}$$

Substitute y value into either of the original equations:

$$3x+18\left(-\frac{1}{3}\right)+7=0$$
$$3x=-1$$
$$x=-\frac{1}{3}$$

So they intersect at $\left(-\frac{1}{3},-\frac{1}{3}\right)$

Try it 1G

1a $(x+2)^2+(y-8)^2=25$

$a=-2$, $b=8 \Rightarrow$ centre is (–2, 8)
Radius is $\sqrt{25}=5$

1b $a=7$, $b=-9$, $r=8$ so equation is
$(x-7)^2+(y-(-9))^2=8^2$

$$(x-7)^2+(y+9)^2=64$$

2a $x^2+y^2-10y+16=0$

Complete the square for y^2-10y:

$x^2+(y-5)^2-25+16=0$

$$x^2+(y-5)^2=9$$

Centre is (0, 5) and radius is $\sqrt{9}=3$

2b $x^2+y^2+6x-12y=0$

Group x terms and y terms:

$x^2+6x+y^2-12y=0$

Complete the square:

$(x+3)^2-9+(y-6)^2-36=0$

$$(x+3)^2+(y-6)^2=45$$

Centre is (–3, 6) and radius is $\sqrt{45}=3\sqrt{5}$

3 Centre is $\left(\frac{4+2}{2},\frac{6+-4}{2}\right)=(3,1)$

Radius is: $r=\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\frac{1}{2}\sqrt{(2-4)^2+(-4-6)^2}$$
$$=\frac{1}{2}\sqrt{(-2)^2+(-10)^2}$$
$$=\sqrt{26}$$
$$(x-3)^2+(y-1)^2=26$$

4a $(x-1)^2+(y+4)^2=50$

Substitute $x=6$, $y=1$:

$(6-1)^2+(1+4)^2=5^2+5^2$
$=50$ so (6, 1) lies on the circle

4b Centre is (1, –4) so gradient of radius to (6, 1) is: $\frac{1--4}{6-1}=\frac{5}{5}$
$=1$

The gradient of the tangent is $m=-1$ since $-1\times1=-1$

Equation of tangent is $y-1=-1(x-6)$

$y=-x+7$